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LETTER TO THE EDITOR

A class of (2+1)-dimensional models with instanton and sphaleron-like solutions

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Abstract. We present a (2+1)-dimensional Skyrme-like model with a symmetry-breaking potential, which in $\mathbb{R}_3$ has charge-n instanton solutions, and in the static limit in $\mathbb{R}_2$ a sphaleron-like solution.

While the quantum tunnelling between topologically distinct vacua of the Weinberg–Salam (gauge–Higgs) field theory is known to be negligible [1], it is possible that at sufficiently high temperatures transitions may occur essentially classically via *sphaleron* field configurations, leading to an appreciable violation of baryon-number conservation. This mechanism was first suggested by Manton [2] and was further developed by Klinkhamer and Manton [3].

Using the sphaleron field configuration of the Weinberg–Salam model, which was previously known as the DHN solution [4], the estimation [5] of the baryon-number violation of electroweak theory can be a task of considerable complexity in quantum theory. For this reason, much attention has been devoted to carrying out this programme employing simplified toy models in lower (than physical) dimensions [6–8]. Notable among these models are those in $1+1$ dimensions, where the sphaleron in question is a constant static solution on S', of the ϕ^4 model and the sine-Gordon model respectively [7]. In the latter example [8], an extended version of the O(3) sigma model in two dimensions has been proposed as the corresponding dynamical system in $1+1$ dimensions. In both these models [7, 8], as also in the original DHN solution on $\mathbb{R}_3$, the sphaleron is an unstable field configuration with finite energy. The energy is the d-dimensional integral of the static field configuration, namely $d=3$ for the DHN case, and $d=1$ for the toy models of [7] and [8]. The sphaleron field's energy is then regarded as the energy barrier between the topologically distinct vacua of the non-static theory. In all these models, the topological charges characterizing the distinct vacua are defined by the usual topological invariant. In the Weinberg–Salam theory, this is taken to be the integral of the Chern–Pontryagin density on $\mathbb{R}_4$, while in the O(3) model of [8], the topological charge is the winding number of the order-parameter field defined on $\mathbb{R}_2$. In both cases, the dynamical models on $d+1$ dimensions, supporting stable instanton field configurations, differ from the dynamical models on d dimensions, which support unstable sphaleron field configurations.

The purpose of the present letter is to propose a new model in 2+1 dimensions, which supports stable instanton field configurations on $\mathbb{R}_3$, and in the static limit supports unstable sphaleron field configurations on $\mathbb{R}_2$. As such, it is an intermediate example with $d=2$ between the DHN case [4] with $d=3$ and the soliton cases [7, 8] with $d=1$. This toy-model aspect, though, is not the main reason for proposing it. Its most important property is that, unlike the $d=1$ and $d=3$ examples discussed above, the instanton *and* sphaleron-like field configurations are supported as solutions by one and the same model.

Having referred to the analogy between our (2+1)-dimensional model, and the (3+1)- and (1+1)-dimensional models [2, 3, 7-9] with sphaleron solutions, we should note that in fact our model bears much closer analogy with the (3+1)-dimensional model [2-4]. This is because in the latter case [2-4] and our case, the static models which have finite energy *unstable* solutions, are defined on $\mathbb{R}_3$ and $\mathbb{R}_2$ respectively. By contrast, in the (1+1)-dimensional cases [7, 8], the static model with unstable solutions is not defined on $\mathbb{R}_1$, but rather on S^1. This is because the latter, be it the ϕ^4 model [7] or the sine-Gordon [8], has only stable solutions on $\mathbb{R}_1$, while the sphaleron solution is required to be unstable. The instability in these cases can be achieved [7] by defining the models in question on S^1 instead of $\mathbb{R}_1$.

To help us arrive at our model, we shall first note a common feature of both the DHN and the extended-O(3) model sphalerons. In each case, respectively in $d=3$ and $d=1$, the scaling properties of the models are consistent with there being finite energy solutions. Such solutions could be topologically stable if there were topological inequalities supplying lower bounds to the energy integrals. In turn, such topological inequalities can be found only for specific field-multiplets defining the dynamical coordinates. Specifically, for the SU(2) Yang–Mill–Higgs model on $\mathbb{R}_3$, such a topological charge (the monopole charge) can be defined if the Higgs field is in the adjoint representation of SU(2), and, for the soliton model in one dimension, such a topological charge (the kink number) can be defined if the field variable consists of one real scalar quantity. The (unstable) sphaleron solutions on the other hand do not occur in the two models just described. Instead in the DHN case [4], the Higgs field is an isospinor and consists of four real components as opposed to the three of an adjoint representation Higgs, and in the extended-O(3) model case [8], the order parameter has two real components as opposed to the single component of the scalar field of the soliton model. In each case ($d=3$ and 1), the additional component of the dynamical field variable serves to parametrize the non-contractible orbit through the instability point.

In the light of these observations, we proceed to consider the model [9] on $\mathbb{R}_2$, $i, j=1, 2$,

$$\hat{\mathscr{L}}_0=\tfrac{1}{2}(\mathrm{i}\partial_{[i}\varphi\partial_{j]}\varphi^*)^2+f(\eta^2-|\varphi|^2, |\partial_i\varphi|^2)+V(\eta^2-|\varphi|^2) \tag{1}$$

where φ is a complex scalar field and η^2 is the (absolute) scale. V is a symmetry breaking potential, and f is a symbolic function representing the quadratic kinetic term $|\partial_i\varphi|^2$. $\hat{\mathscr{L}}_0$ is regarded as the static limit of a Lagrangian $\mathscr{L}$ in 2+1 dimensions.

It was shown in [9] that subject to the asymptotic condition

$$|\varphi|^2 \xrightarrow[|x|\to\infty]{} \eta^2 \tag{2}$$

the volume integral of (1) is minimized by topologically stable field configurations, by virtue of the topological inequality

$$\int \hat{\mathscr{L}}_0\,\mathrm{d}^2x \geqslant 2\mathrm{i}\varepsilon_{ij}\int \sqrt{V}\,\partial_i\varphi\partial_j\varphi^*\,\mathrm{d}^2x. \tag{3}$$

Following our above descriptions of the $d=3$ and $d=1$ sphalerons, we modify the model (1) by augmenting the dynamical coordinate φ with an additional component ϕ_3. Thus, in place of $\varphi=\phi_1+\mathrm{i}\phi_2$, our new field variable is $\Phi=\boldsymbol{\phi}\cdot\boldsymbol{\sigma}$ in terms of the Pauli spin matrices $\boldsymbol{\sigma}$. This yields

$$\mathscr{L}_0=-\tfrac{1}{2}\operatorname{tr}\Phi_{ij}^2+f(\eta^2-\Phi^2,\Phi_i^2)+V(\eta^2-\Phi^2) \tag{4}$$

where we use the notation $\Phi_i:=\partial_i\Phi$ and $\Phi_{ij}:=[\Phi_i,\Phi_j]$. Again f is a symbolic function representing the, now non-Abelian, quadratic kinetic term Φ_i^2. One should note that the scaling properties of the integral of (4) over $\mathbb{R}_2$, are still consistent with the existence of finite energy solutions, but now we have lost the topological inequality (3). This is so because the corresponding topological charge density $\varepsilon_{ij}\operatorname{tr}\sqrt{V}\,\partial_i\Phi\partial_j\Phi$ can be seen *not* to be a total divergence, in contrast to the density on the right-hand side of (3) defined in terms of the complex field φ. As a consequence, we would expect any finite energy solutions to the equations of motion that may be found to be unstable. But this is precisely what would be expected of a sphaleron field, especially if we remember that the source of this new instability is the additional component of the multiplet Φ, over and above the number of degrees of freedom of the old field φ in (1). We adopt (4) therefore as the static version of a candidate for a (2+1)-dimensional model with instanton *and* sphaleron-like solutions, and proceed to verify these properties. For technical reasons, we consider the instanton properties first.

Instantons. It is useful to specialize the Lagrangian (4), considered on $\mathbb{R}_3$, to analyse the stability of the instanton solutions. This problem was considered in some detail, and analysed in [10]. To avoid the ubiquity of models afforded by the symbolic functions f and V in (4), we specialize to some specific choices of these functions.

To start with, according to the virial theorem or scaling argument, it is necessary to keep only the first and second, or the first and third, terms in

$$\mathscr{L}=-\tfrac{1}{2}\operatorname{tr}\Phi_{\mu\nu}^2+f(\eta^2-\Phi^2,\Phi_\mu^2)+V(\eta^2-\Phi^2) \tag{5}$$

to enable finite action solutions on $\mathbb{R}_3$. Here $\mu=1,2,3$ labels the coordinate x_μ of $\mathbb{R}_3$. However, as explained in detail in [10, 11], in the absence of the second term f, topological stability would dictate the inclusion of an additional sextic kinetic term $\Phi_{\mu\nu\rho}^2$ which we wish to avoid here. We therefore must retain the second term f in (5). Topological stability does not demand the presence of third term V. Nevertheless, we shall retain V, in anticipation of a similar scaling argument, for the static Lagrangian $\mathscr{L}_0$ of (4), in $\mathbb{R}_2$. Retaining both f and V in (5), we opt to specialize (5) to the simplest sub-model arising from the direct descent from the eight-dimensional conformally invariant generalized Yang–Mills system [11]. The distinguishing feature of this model, other than its relative simplicity, is that it involves no dimensional constants apart from the constant η setting the scale of the field Φ. Our choice is

$$\mathscr{L}=-\tfrac{1}{2}\operatorname{tr}\Phi_{\mu\nu}^2+\tfrac{1}{2}\operatorname{tr}\{S,\Phi_\mu\}^2+\operatorname{tr}S^4 \tag{5$'$}$$

where $S:=\eta^2-\Phi^2$ and $\{,\}$ means anticommutation. We stress that our choices for the symbolic function f and of V in (6) are not unique.

The only term in (5) in whose definition we have no freedom is the quartic kinetic term $\Phi_{\mu\nu}^2$. For both the other terms f and V, namely the quadratic kinetic term and the potential respectively, there is considerable freedom in their definitions. The only constraint in exercising this freedom is the necessity to obtain a topological inequality like (7) below. The specific choice (5$'$) is privileged only in that it happens to be the model *directly* arrived at by descending from the corresponding higher-dimensional gauge field model [11], and as a result involves no other dimensional constants, apart from the absolute scale η. Otherwise, for the simplest version of (5), one can choose

$$f=\lambda_1\Phi_\mu^2 \qquad V=\lambda_2(\eta^2-\Phi^2)^2 \tag{6}$$

where λ_1, λ_2 are some (coupling) constants with appropriate dimensions.

Both (5′), and the singlet model obtained by the choice (6) in (5), have the additional symmetry $\Phi\to g\Phi g^{-1}$, where g is a rigid SU(2) transformation as well as under the discrete symmetry $\Phi\to-\Phi$. These symmetries would definitely have to be taken into account in the quantization of these systems, but this does not form part of our considerations here. We refer to these symmetries here, and especially the discrete one, because it is of some consequence below, in characterizing the static sphaleron-like solutions in $\mathbb{R}_2$.

Returning to the model (5′), the topological stability of the instanton is then a consequence of the inequality

$$\operatorname{tr}\left(\mathrm{i}\Phi_{\mu\nu}-\frac{1}{\sqrt{2}}\,\varepsilon_{\mu\nu\rho}\{S,\Phi_\rho\}\right)^2\geqslant 0. \tag{7}$$

Adding the positive-definite term $2\operatorname{tr}S^4$ to the left-hand side of (7) without disturbing the inequality, and expanding (7), we have

$$\mathscr{L}\geqslant 2\sqrt{2}\,\mathrm{i}\varepsilon_{\mu\nu\rho}\operatorname{tr}\{S,\Phi_\mu\}\Phi_\nu\Phi_\rho \tag{8}$$

the right-hand side of which can be shown to be a total divergence [10, 11], whose integral, subject to the asymptotic condition

$$\operatorname{tr}\Phi^2\xrightarrow[|x|\to\infty]{}\eta^2 \tag{9}$$

guarantees a non-zero lower bound for the action which is proportional to a winding number n. Thus the model (6) is endowed with a stable instanton field configuration in $\mathbb{R}_3$.

Since the instanton field configurations of this (and other) model(s) on $\mathbb{R}_3$ were discussed in some detail in [10], it suffices here to recall that these instantons correspond to *topologically distinct vacua* characterized by a winding number n, which in this case is the topological charge given (up to normalization) by the integral of the right-hand side of (8). The n-dependence of these field configurations is given by [10]

$$\begin{aligned}\phi_1&=\phi(R)\sin\theta\cos n\varphi\\ \phi_2&=\phi(R)\sin\theta\sin n\varphi\\ \phi_3&=\phi(R)\cos\theta\end{aligned} \tag{10}$$

where $R=\sqrt{x_\mu x_\mu}$, θ and φ are the polar and azimuthal angles in three dimensions, and ϕ_μ defines $\Phi=\phi_\mu\sigma_\mu$, θ and φ parametrize both the field and the space $S^2\subset R_3$.

All above considerations (7)-(10) can be made even more simply for the choice of f and V in (5), given by (6).

Sphaleron-like solution. The static version of (5′), defined on $\mathbb{R}_2$,

$$\mathscr{L}_0=-\tfrac{1}{2}\operatorname{tr}\Phi_{ij}^2+\tfrac{1}{2}\operatorname{tr}\{S,\Phi_i\}^2+\operatorname{tr}S^4 \tag{11}$$

will now be shown to have a sphaleron-like solution. First we recall that, according to the scaling argument, the equations of motion for (11) can have finite energy solutions irrespective of the absence/presence of the second term quadratic in Φ_i. We also note that now we have no topological inequality analogous to (8), so that the finite energy configurations are non-topological.

We consider the following ansatz for the (unstable) sphaleron-like field configuration

$$\Phi = \sigma_1 \eta f(r) \sin\mu \cos n\varphi + \sigma_2 \eta f(r) \sin\mu \sin n\varphi + \sigma_3 \eta g(r) \cos\mu \tag{12}$$

where $r^2 = x_i x_i$ $(i = 1, 2)$, while μ is a constant which causes the instability of the energy integral (15) below. As such, this parameter μ is analogous to the angular parameter which characterizes the non-contactible loop (NCL) of the DHN sphaleron [2, 3]. It is however different from the corresponding parameter of the DHN case, because our static field (12) is not a NCL. This is so because $\Phi^\infty(\mu = 0) = \eta\sigma_3$ (since $g^\infty = 1$, cf (21) below), while $\Phi^\infty(\mu = \pi) = -\eta\sigma_3$. That is, as μ varies between 0 and π the vacuum Φ^∞ does not go back to its original value. The reason that this happens is because of the discrete symmetry $\Phi \to -\Phi$ referred to above. Thus we do not have a NCL in (12), and for this reason, we refer to this solution only as a sphaleron-like solution.

Unlike the NCL of the DHN sphaleron [2, 3], which represents the path between any vacuum and itself, our sphaleron-like solution represents the path between two distinct vacua $\Phi^\infty(0)$ and $\Phi^\infty(\pi) = -\Phi^\infty(0)$. Nevertheless, we hope that this solution is physically relevant since for $\mu = \pi/2$ it corresponds to the top of the energy barrier between these vacua. The actual sphaleron-like field (12) with $\mu = \pi/2$ is an unstable solution, since it maximizes the energy functional. Before proceeding to demonstrate this instability, we must check the consistency of this ansatz. This involves the verification that the Euler–Lagrange equations of the system (11) on $\mathbb{R}_2$

$$\partial_i[\Phi_j, \Phi_{ij}] + \tfrac{1}{2}\partial_i\{\{S, \Phi_i\}, S\} = 2\{\Phi, S^3\} - \{\Phi, \{\Phi_i\{\Phi_i, S\}\}\} \tag{13}$$

for the field configuration (12), are solved by the Euler–Lagrange equations for the one-dimensional subsystem with Lagrangian $L[f, g]$, defined by $S = \int \mathscr{L} r \, \mathrm{d}r \, \mathrm{d}\varphi \equiv 2\pi \int L \, \mathrm{d}r$, or

$$L[f(r), f'(r); g(r), g'(r)] \equiv 2\pi r \mathscr{L}[f, f'; g, g'] \tag{14}$$

in terms of the coordinates f, g and their 'velocities' $f' \equiv \mathrm{d}f/\mathrm{d}r$ and g'. This is a very straightforward if tedious task, and we limit ourselves to stating that indeed the Euler–Lagrange equations arising from the variations of $f(r)$ and $g(r)$, respectively, for (14), solve the equations (13) for the field configuration (12). These equations are rather lengthy expressions, and are not recorded here, but we confirm that the ansatz (12) is *consistent*.

The existence [12] of the sphaleron field configuration (12) then follows from the positive definiteness of the energy integral

$$\begin{aligned} E[f, g, \mu] = 4\pi \int_0^\infty \bigg\{ & 4\eta^4 \sin^2\mu \frac{f^2}{r} (g'^2 \cos^2\mu + f'^2 \sin^2\mu) \\ & + 2\eta^6 r[1 - (g^2 \cos^2\mu + f^2 \sin^2\mu)]^2 \\ & \times \left[(g'^2 \cos^2\mu + f'^2 \sin^2\mu) + \frac{f^2}{r^2} \sin^2\mu \right] \\ & + \eta^8 r[1 - (g^2 \cos^2\mu + f^2 \sin^2\mu)]^4 \bigg\} \, \mathrm{d}r. \end{aligned} \tag{15}$$

An intuitive demonstration of the fact that the energy functional (15) takes on its maximum value at $\mu=\pi/2$ can be given as follows. If we set $f(r)=g(r)$ in (15), all the $\cos^2\mu$ terms disappear, and the functional $E[f,g,\mu]$ depends on μ only through $\sin^2\mu$. Then it is clear that $E[f,g,\pi/2]$ is the maximum value of E. This field configuration, however, is not a solution, as setting $f=g$ renders the ansatz (12) inconsistent. We expect, however, that for the actual solutions, this conclusion remains true. Strictly, a numerical verification here is in order, since we do not have $g(r)$ and $f(r)$ explicitly. We intend to perform this elsewhere, together with a more extensive analysis of lower dimensional $(d<3+1)$ models.

There is, admittedly, a certain lack of rigour in the above argument for the instability of the solutions (12). It is clear from the energy integral (15) that for $\mu=0$ (and π), the first term, which is the contribution of the quartic kinetic energy, vanishes. The second term however, which is the contribution of the quadratic kinetic energy, does not vanish, but only loses one of its (positive definite) constituents. The rest, the potential energy contribution, is insensitive to the angle μ. That the solution (12) is unstable owing to the μ-dependence of (15), is not in question. What is strictly required however is that at $\mu=0$ (and π), the energy integral should actually vanish. This we have not succeeded in showing rigorously, but see no obstacle to such a demonstration. In this respect, the integral (15) is exactly on the same footing as the energy integral of the DHN model, equation (29) of [12]. The vanishing of this integral was argued by Burzlaff [12], and is responsible for the instability of the DHN solution. Correspondingly, our energy integral (15), as also the integral (29) of [12], would take on its maximum value for $\mu=\pi/2$, as argued above.

Topological charges. We have shown above that the $(2+1)$-dimensional model given by the Lagrangian (6) is endowed with charge-n instanton solutions in $\mathbb{R}_3$, and its static version (10) with a sphaleron solution in $\mathbb{R}_2$. As the latter is expected to be the energy barrier given by the static fields, between the topologically distinct vacua of the same model in $\mathbb{R}_3$, it remains for us to demonstrate this property by verifying that the (topological) charge integral

$$q\equiv\int \mathrm{d}t\,\mathrm{d}^2x\,\rho=2\sqrt{2}\,\mathrm{i}\varepsilon_{\mu\nu\rho}\int_{t=-\infty}^{+\infty}\int_{\mathbb{R}_2}\mathrm{tr}\,S\Phi_\mu\Phi_\nu\Phi_\rho\,\mathrm{d}t\,\mathrm{d}^2x \tag{16}$$

(cf (8)) for a $(2+1)$-dimensional field configuration including the sphaleron field (12), can be evaluated as a surface integral whose value is controlled by the topological properties of the field Φ, in $\mathbb{R}_2$. To this end, we follow the procedure first suggested in [2, 3], and employed in [13]. This involves adopting a field configuration $\Phi(x,t)$ given by (12), where the functions f and g depend on the radial variable r of $\mathbb{R}_2$, but where the coordinate μ is taken to be a function of t, $\mu=\mu(t)$. Writing $\mathrm{d}\mu/\mathrm{d}t\equiv\dot{\mu}$, the integral (16)

$$q\approx\int\mathrm{d}t\,\mathrm{d}^2x\,\rho\approx\frac{4}{\sqrt{2}}\mathrm{i}\varepsilon_{ij}\,\mathrm{tr}\int\int\mathrm{d}t\,\mathrm{d}^2x\,S(\Phi_t\Phi_i\Phi_j+\Phi_i\Phi_t\Phi_j+\Phi_i\Phi_j\Phi_t) \tag{17}$$

can then be expressed as

$$q=q_0+q_1 \tag{18}$$

$$q_0\propto 2\pi\int r\,\mathrm{d}r\int\mathrm{d}t\,\dot{\mu}\sin\mu\frac{f}{r}[gf'+(fg'-gf')\cos^2\mu] \tag{18a}$$

$$q_1 = 2\pi \int r\,\mathrm{d}r \int \mathrm{d}t\,\dot{\mu}\sin\mu \frac{f}{r}[(g^2-f^2)\cos^2\mu + f]$$
$$\times[(fg'-gf')\cos^2\mu + gf']. \tag{18b}$$

Now allowing $\mu(t)$ to vary between 0 and π as t varies from $-\infty$ to $+\infty$, we can perform the integrals (18*a*, *b*) as integrals with respect to $\cos\mu$, between the limits $\cos\mu(t=\pm\infty)=\pm1$. The result is

$$q_0 = 4\pi\int_0^\infty h_0(r)\,\mathrm{d}r \tag{19a}$$

$$q_1 = 4\pi\int_0^\infty h_1(r)\,\mathrm{d}r \tag{19b}$$

where both integrals can be evaluated simply by using the topologically meaningful boundary values of f and g, by virtue of the fact that the functions h_0 and h_1 are given as the derivatives

$$h_0 = \frac{1}{3}\frac{\mathrm{d}}{\mathrm{d}r}(gf^2) \tag{20a}$$

$$h_1 = \frac{1}{15}\frac{\mathrm{d}}{\mathrm{d}r}[gf^2(g^2+2f^2)]. \tag{20b}$$

The integrals (19*a*, *b*) are then immediately evaluated using the asymptotic conditions

$$g(\infty) = f(\infty) = 1 \tag{21}$$

which is consistent with the finite-energy condition

$$\operatorname{tr}\Phi^2 \xrightarrow[r\to\infty]{} \eta^2 \tag{22}$$

for the field (12), analogous to the finite-action condition (9), for the field (10). The boundary condition at the origin of r is

$$f(0) = 0 \tag{23}$$

which is also the necessary condition for the single-valuedness of the field (12). This defines, up to normalization, the topological charge of the (2+1)-dimensional model (6), which has charge-n instanton solutions (10) in $\mathbb{R}_3$, and in the static limit a sphaleron solution (12) in $\mathbb{R}_2$. Thus one can associate a finite value of the instanton (topological) charge with the (non-topological) sphaleron.

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